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ANALYTICAL AND STATISTICAL ASSESSMENTS OF THE METHODOLOGY FOR EVALUATING THE QUALITY OF LAMINATED MAGNETIC CORES

The paper solves the relevant issue of improving the reliability of electromechanical converters with a steel laminated magnetic core by implementing effective measures to diagnose the quality of the laminated core. A method for assessing the quality of laminated magnetic cores with insulated sheets is proposed. The method is based on the analysis of the response of electromagnetic interconnected circuits to test transient processes in order to assess the development of defects in the inter-sheet insulation and the concentration of parasitic eddy currents associated with specific losses in the magnetic core which is the normative method of magnetic core evaluation that is closest to the operation of an electric machine under real conditions. Taking into account the complexity and labour intensity of regulated methods of core quality control, a reasonable assessment of the relationship between the eddy current time constant and the specific losses in the magnetic circuit will significantly simplify the test methodology, the implementation of which does not require complex and expensive equipment and high qualification of the service personnel. In addition to the physical connection, the paper shows the statistical relationship between the eddy current time constant and the specific losses in a number of real magnetic circuits and performs statistical processing of the experimental data. A linear regression dependence is obtained and the correlation coefficient is estimated, which confirms a significant relationship between the above components on the example of magnetic cores of different geometries. Confidence intervals for defect-free and defective magnetic cores are estimated, taking into account the geometric dimensions, power, and number of poles of the studied electric motors.

Keywords: *Magnetically coupled circuits, damping time constant, specific losses in magnetic circuit, reliability, remaining insulation resource, laminated magnetic cores*

Introduction

Electromechanical and electromagnetic energy converters use electrical steels with high magnetic conductivity and low magnetisation losses as magnetic flux conductors. In the process of cyclic changes in the magnetic structure, processes associated with hysteresis losses and eddy current losses occur. At industrial frequencies of 50-60 Hz, the hysteresis loss component is dominant. Depending on the steel grade, the predominance of hysteresis losses over eddy current losses at industrial frequencies ranges from three to six times. However, in technological and operational processes, the component of eddy current losses, due to deterioration of the inter-sheet insulation, can significantly exceed hysteresis losses. Methods for assessing the quality of magnetic cores traditionally use the dependence of the quality of the magnetic core on the specific losses in steel at induction close to machine operation and industrial frequency. These methods are standardised, but they are labour-intensive and do not meet modern requirements for the efficiency of control tests. Therefore, there is a need to develop methods for more effective control methods that can be associated with regulatory tests. It is proposed to use as a diagnostic feature the relationship between the time constant of eddy currents in a magnetic circuit and the specific losses that meet the regulated methods.

The purpose of the study is to establish a physical and statistical relationship between the specific losses in a magnetic core at a fixed induction and frequency and the time constant of eddy currents for magnetic cores of different geometries.

The objective of the study is to investigate the physical connection of the fast process of establishing magnetic flux in magnetically coupled circuits with the reaction of the secondary system in the form of eddy current circuits in the defect-free and defective level of the laminated magnetic core.

Main Material of the Study

Physical bases of connection between parameters of transient processes and specific losses in laminated magnetic cores

If a test coil is wound around the back of the magnetic core, it will also induce an EMF. A single sheet can be considered as an element in a composite magnetic core, if the insulation between the sheets is of high quality. Let us consider the distribution of EMF, currents and eddy current losses in the cross-section of the sheet under alternating magnetic flux in the sheet. We assume that the equipotential lines of the elementary circuit are symmetrical relative to the centre. As the circuit grows, its cross-section q_x increases according to the quadratic law, and the length of the circuit l_x – The induced EMF will depend on the rate of change of the flow. Using the operator p for ease of writing, the following relations can be obtained for a circuit with a width of $2x$:

$$e_x = p\Phi_x = pBq_x \tag{1}$$

Where $q_x = 2x^2\frac{h}{b}$ – cross-section of an axisymmetric contour, h – sheet height.

Resistance of a contour with a thickness Δx when section length $l = 1$:

$$\Delta r_x = 2\left(2x + \frac{h}{b}x\right)\frac{\rho_{ст}}{\Delta x} \tag{2}$$

Where ρ_s – specific losses of steel. The elementary current in the considered circuit will be:

$$\Delta i_x = \frac{e_x}{\Delta r_x} = \frac{pB}{\rho_s} \tag{3}$$

This is true for the case when $h >> b$. When $B = B_m sin\omega t$ і $pB = 2\pi f B_m$ the current value of the EMF will be:

$$E_x = \pi\sqrt{2}fBq_x \tag{4}$$

It is obvious that when the induction decreases due to the demagnetising effect of parasitic eddy current circuits, the effective EMF value will decrease with increasing frequency.

The losses in an elementary circuit of length $l = 1$ will be:

$$\Delta p_x = \frac{E_x^2}{\Delta r_x} = \frac{\left(\pi\sqrt{2}fB_m 2\frac{h}{b}x^2\right)^2}{2\frac{h}{b}x\rho}\Delta x \tag{5}$$

Total eddy current losses:

$$P_{ec} = \int_0^b dp_x = \frac{2\pi^2 f^2 B_m^2 2\frac{h}{b}}{\rho}\int_0^b x^3 dx = \frac{\pi^2 f^2 B_m^2}{\rho}hb^3 \tag{6}$$

Specific eddy current losses:

$$\mathrm{P}_{spec.ec} = \frac{P_{в}}{m} = \frac{\pi^2 f^2 B_m^2 b^2}{2\rho g_s} \tag{7}$$

Where $m = 2hblg_s$ – considered element mass, g_s – steel density $7{,}8 * 10^3\ \frac{kg}{m^3}$. $\mathrm{P}_{spec.ec}$ is proportional to the product $f^2B_m^2$, a constant coefficient in this formula $k_{ec} = \frac{\pi^2 b^2}{2\rho g_s}$.

Thus, for a steel sheet with $b = 0{,}25mm$ and $\rho = 2 * 10^{-7}$ losses are $\mathrm{P}_{spec.ec} = 0{,}5\ \frac{W}{kg}$, which fully corresponds to the table data for electrical steel. Hysteresis losses are several times higher. For 2013 steel, the total losses:

$$(\mathrm{P}_{spec.ec} + \mathrm{P}_{spec.h}) = 0{,}5 + 2 = 2{,}5\ \frac{W}{kg} \tag{8}$$

In other words, in order for the total losses to increase significantly, the losses due to eddy currents caused by defects in the interlayer insulation must increase at least by several times.

For a defective magnetic core, assuming a uniform distribution of defects in the interlayer insulation, in addition to eddy currents that are confined to individual sheets, there are common integral currents that cover the entire package. With a uniform distribution of defects, the processes in the package can be described in the same way as in a single sheet, but it is necessary to take into account electrical anisotropy, i.e. the fact that due to defects, the specific resistance in the transverse direction is much higher than the resistance of steel [1].

The $\frac{h}{b}$ ratio can be any, more often less than one. The value of b in this case plays the role of the width of the entire package, i.e. if the thickness of the sheet was equal to the width of the package.

According to (1) EMF of xth circuit will be:

$$e_x = pB\frac{2hx^2}{b} \tag{9}$$

And the resistance of the elementary circuit according to (2) will be:

$$\Delta r_x = \frac{2\left(2x\rho_{def} + \frac{h}{b}x\rho_s\right)}{\Delta x} = \frac{4x\rho_{def}}{\Delta x} \tag{10}$$

where ρ_{def}– electrical resistance of the defective circuit.

The current in the elementary circuit according to (3) will be:

$$\Delta i_x = \frac{pB\frac{h}{b}}{2\rho_{def}}x\Delta x \tag{11}$$

$$\Phi_x = \left(\frac{1}{x}\int B_x dx\right)q_x = \frac{pB_x h^2 \mu x^2\left(b^2 - \frac{x^2}{3}\right)}{2\rho_{def}b^2} \tag{12}$$

For alternating magnetic flux, eddy current losses are determined by the following equation:

$$\Delta p_x = \frac{E_x^2}{\Delta r_x} = \frac{\left(\pi\sqrt{2}fB_m 2\frac{h}{b}x^2\right)^2}{4\rho_{def}x} \quad (13)$$

The eddy current losses in a magnetic circuit with integral defects will be as follows:

$$P_{ec} = \int_0^b \quad dp_x = \frac{\pi^2 2f^2 B_m^2 \left(\frac{h}{b}\right)^2}{\rho_{def}} \frac{b^4}{4} = \frac{\pi^2 f^2 B_m^2 (hb)^2}{2\rho_{def}} \quad (14)$$

$$P_{spec.ec} = \frac{P_{ec}}{m} = \frac{\pi^2 f^2 B_m^2 hb}{2\rho_{def} g_s} = k_{ec.def.} f^2 B_m^2 \quad (15)$$

$$k_{ec.def.} = \frac{\pi^2 hb}{2\rho_{def} g_s} \quad (16)$$

As can be seen, the relationship between the specific losses for eddy currents in a magnetic core with integral defects is direct through $k_{ec.def}$ which depends on the cross-section of the magnetic core $\frac{h}{b}$ and the resistance of the defective circuits ρ_{def}.

In order for the total specific losses to increase compared to a defect-free magnetic core, for example, by a factor of 2 (up to 5 W/kg), different degrees of defectiveness are required for magnetic cores of different geometries.

To do this, we define ρ_{def}:

$$\rho_{def} = \frac{\pi^2 f^2 B_m^2 hb}{2 g_s P_{spec.ec}} = 0{,}63hb \quad (17)$$

For the magnetic core of a small machine, $hb = 0{,}01 * 0{,}1 = 0{,}001\ m^2$, $\rho_{def} = 6{,}3 * 10^{-4}\ Ohm * m$. As the cross-section increases, the same effect will be achieved at higher ρ_{def}, a and the total losses in steel increase in proportion to the square of the cross-section.

The inertia and demagnetising effect of the defective circuits is superimposed on the magnetic inertia and the reduction of induction in the magnetic core itself. The decrease of the average induction value will occur faster in defective magnetic circuits compared to defect-free ones with increasing frequency. Relative to the coil, which creates a magnetic field in the magnetic core, the total resistance Z will be determined [2]:

$$Z = \frac{U}{I} = \frac{j}{1+j} \cdot \frac{adh\omega\mu}{lb} N^2 th(1+j)\frac{b}{a} \quad (18)$$

Analysing the above expressions, it can be concluded that the level of defectiveness of a magnetic core can be determined through the same parameters inherent in the eddy current time constant and specific losses of a magnetic core at an industrial frequency and a fixed induction, which is usually 1 T. For defect-free magnetic cores, the relationship between T_{ec} and $P_{spec.ec}$ is made through the square of the sheet thickness, electrical conductivity and magnetic permeability of the steel material. For a defective magnetic core, a parasitic circuit is superimposed on the eddy current circuit that occurs in a defect-free magnetic core, which depends on the degree of defective metal overlaps and the overall geometric component of such circuit, which is proportional to the product hb (18).

Based on the proposed method of quality control of the magnetic cores, model installations were created, which allowed to conduct large-scale experiments at the enterprises ‘Miskvodokanal’ of Sumy City Council and ENERSIS UKRAINE LLC company.

The experiment was carried out on a large number of magnetic cores of IMs with power from 0,37 kW to 37 kW with different degrees of defects. The magnetic losses in the cores were measured by the wattmeter method at an induction of 1 T and the time constant of eddy currents T_{ec} μs. The results are presented in table 1. The experiment included testing more than 200 magnetic cores of different quality and geometry, which corresponds to a range of motor park capacities available at the enterprises.

As samples for the experiment, both magnetic cores that were not in operation and magnetic cores that are in operation, as well as magnetic cores that have undergone a repeated technological cycle of heating in a furnace to a temperature of 350-450 ℃ in order to burn out the old winding before rewinding electric motors, were used. Thus, there were two groups of magnetic cores: high-quality and defective, with varying degrees of damage to the interlayer insulation.

Identification of the correlation coefficient and linear regression analysis of samples on the relationship between specific losses and the generalised diagnostic parameter

To perform such an analysis, a fully automated calculation was used for the largest sample of $N = 200$ to represent the relationship between μ_x and p_{spec} for magnetic cores from the experiments shown in table 1, where μ_x – is the mean value of a random variable. In this table, the values of specific losses were measured at an induction of 1 T and industrial frequency are given in W/kg.

Correlation analysis allows to determine the degree of relationship between two or more variables. However, it is also desirable to have a model of this relationship that would make it possible to predict the value of one random variable from the specific values of another.

In our case, the correlation analysis of the data from numerous experiments established a significant linear relationship between the diagnostic parameters in the magnetic cores under study and the specific magnetisation losses. The logical next step is to specify this relationship so that this generalised diagnostic parameter could be used to predict the specific losses P_{spec}.

Table 1 - Relationship of specific losses in magnetic cores of different defect degrees with T_{ec} [3]

Pspec	3,28	3,49	3,44	3,28	3,62	3,33	3,62	4,1	4,25	4,36	4,92
Tec	42	43	48	43	48	45	47	51	53	51	49
Pspec	5,12	5,39	5,62	5,68	5,36	4,38	4,92	5,65	5,34	5,46	5,11
Tec	53	54	59	66	52	48	51	59	57	58	64
Pspec	5,62	4,80	4,92	4,75	5,35	5,55	5,98	6,32	6,45	6,0	6,13
Tec	62	55	53	52	55	58	71	75	81	72	87
Pspec	6,20	6,18	6,4	6,45	6,31	6,4	6,12	6,53	6,27	6,63	6,84
Tec	88	86	91	86	83	84	87	78	82	85	92
Pspec	6,8	6,32	6,55	6,27	6,35	7,14	7,45	7,05	6,94	6,40	6,4
Tec	91	74	79	84	85	97	91	99	89	77	83
Pspec	6,78	7,0	7,2	7,24	7,55	7,46	7,9	8,08	7,9	7,46	7,93
Tec	100	107	113	108	102	101	115	119	123	100	128
Pspec	8,0	8,21	8,36	8,64	7,94	8,2	8,32	8,6	8,2	8,56	9,0
Tec	104	119	121	133	107	116	135	129	132	139	120
Pspec	9,60	9,21	9,36	8,84	8,05	7,68	8,63	8,43	8,02	8,43	8,68
Tec	130	128	126	117	125	111	134	119	124	127	117
Pspec	8,9	8,45	9,63	9,2	9,40	10,8	8,72	9,0	9,2	9,34	8,63
Tec	138	132	130	132	138	152	108	114	124	128	136
Pspec	9,8	9,64	9,55	9,84	9,39	9,69	10,12	9,87	11,0	9,85	10,16
Tec	152	142	141	144	138	135	157	148	154	152	158
Pspec	10,28	11,42	11,23	11,0	11,52	10,63	10,05	10,48	10,69	11,57	11,23
Tec	160	150	145	172	174	161	139	144	152	181	175
Pspec	11,10	10,88	10,67	10,58	10,85	11,05	11,45	11,38	10,93	11,5	11,43
Tec	172	170	163	158	147	171	173	176	169	177	179
Pspec	11,89	11,71	11,5	11,25	11,8	11,34	11,02	11,47	11,25	10,96	10,74
Tec	186	181	176	174	180	177	166	168	174	169	166
Pspec	11,43	11,28	12,3	11,48	11,3	11,85	12,05	11,68	11,55	10,96	11,8
Tec	181	174	180	175	172	178	182	172	177	159	172
Pspec	11,98	11,68	11,35	11,8	11,87	12,3	11,45	11,36	11,8	11,2	12,3
Tec	188	182	180	173	184	185	178	172	173	180	185
Pspec	11,93	11,24	11,67	11,7	12,4	12,38	12,87	11,45	12,39	12,12	12,43
Tec	188	169	174	179	190	184	192	182	185	180	183
Pspec	12,66	11,96	12,12	12,7	13,01	12,75	11,96	13,56	12,88	13,4	13,27
Tec	191	189	192	190	200	197	183	214	207	211	209
Pspec	12,63	13,78	13,07	14,54	13,4	13,25	12,75	12,4	12,36	12,89	13,15
Tec	194	203	209	218	205	211	199	202	201	212	215
Pspec	13,87	13,47									
Tec	210	209									

Methods for solving such problems are called regression analysis [5]. In our case, x can be a diagnostic parameter, and y – can be specific losses. The linear relationship between two random variables means that the forecast of the value of y based on this analysis is as follows:

$$\tilde{y} = A + Bx \qquad (19)$$

where A and B are, respectively, the segment of the ordinate axis that forms the line and its slope. If the data are related by a perfect linear relationship $r_{xy} = 1$, then the predicted value of $\tilde{y}_i$ will be exactly equal to the observed value of y_i for any given x_i.

However, in practice, there is usually no perfect linear relationship between the data. Nevertheless, assuming a linear relationship and an unlimited sample, it is possible to choose such a value of A and B, that will allow to predict the expected value of y_i for any given x_i. This means that $\tilde{y}_i$ does not necessarily coincide with the observed value of y_i, corresponding to a given x_i, but it will be equal to the average of all such values.

The generally accepted procedure for determining the coefficients of equation (19) is to choose such values of A and B, that minimise the sum of the squared deviations of the values from the predicted value of y. This approach is called the least squares method [5].

Using the data in table 1, let's define a line that gives a linear prediction of the specific losses in the magnetic core P_{spec} by the diagnostic parameter T_{ec}. It should be noted that regression in general has a more complex relationship than a linear one, but since the correlation coefficient is large, we can use a linear regression function. As before, let x be the diagnostic parameter, and $y = P_{spec}$.

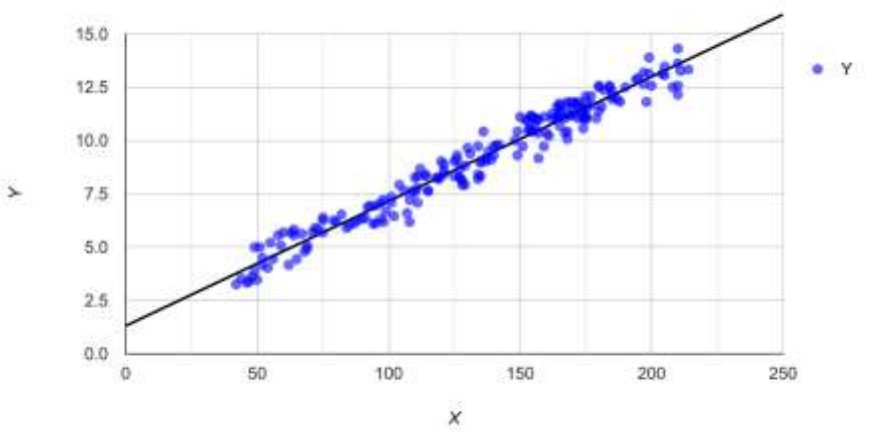

Figure 1 – *Experimental data and regression line*

Accordingly, the regression line that estimates the average specific losses by the diagnostic parameter T_{ec} is as follows:

$$y = 1{,}3135 + 0{,}05838x \tag{20}$$

and is shown in figure 1.

Based on it for $\mathrm{T}_{ec} = 50\ \mu s$ we get $P_{spec} = 4{,}23\ \frac{W}{kg}$; for $\mathrm{T}_{ec} = 80\ \mu s$ we obtain $P_{spec} = 5{,}98\ \frac{W}{kg}$; for $\mathrm{T}_{ec} = 130\ \mu s$ we get $P_{spec} = 8{,}9\ \frac{W}{kg}$; for $\mathrm{T}_{ec} = 180\ \mu s$ we obtain $P_{spec} = 11{,}82\ \frac{W}{kg}$.

Having calculated the confidence interval and prediction interval, we obtained the graph in figure 2.

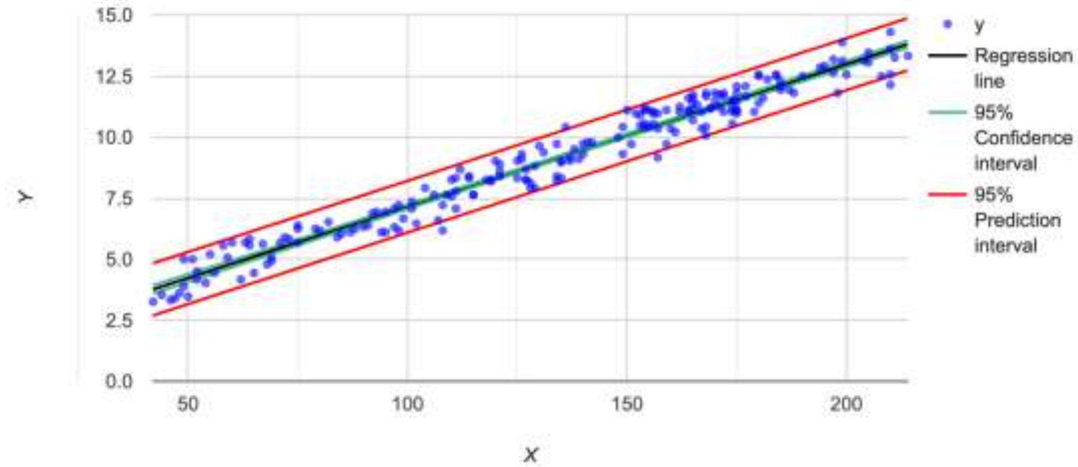

Figure 2 – *Experimental data, regression line, prediction interval and confidence interval*

Thus, according to the results of correlation and regression analysis of the presented sample, it can be stated that for the diagnostic parameter $\mathrm{T}_{ec} = 50\ \mu s$ specific losses according to confidence interval are in the range from 4,05 to 4,37 W/kg, and according prediction interval are in the range from 3,14 to 5,28 W/kg; for $\mathrm{T}_{ec} = 80\ \mu s$ specific losses according to confidence interval are in the range from 5,88 to 6,22 W/kg, and according prediction interval are in the range from 4,93 to 7,06 W/kg; for $\mathrm{T}_{ec} = 130\ \mu s$ specific losses according to confidence interval are in the range from 8,82 to 8,97 W/kg, and according prediction interval are in the range from 7,83 to 9,95 W/kg; for $\mathrm{T}_{ec} = 180\ \mu s$ specific losses according to confidence interval are in the range from 11,68 дtoo 11,89 W/kg, and according prediction interval are in the range from 10,73 to 12,85 W/kg.

The correlation coefficient R is 0,98, which means that there is a very strong, direct dependence of the specific losses on the eddy current time constant T_{ec}. And the coefficient of determination R^2 is 0,961, which means that 96,1% of the variability of the specific losses is explained by T_{ec}.

Selection by the relationship between the time constant of eddy currents and geometrical dimensions for high-quality magnetic circuits

Table 2 shows a selection of the relationship between the time constant of eddy currents T_{ec} in magnetic cores at fixed P_{spec} and different geometries of high-quality charge magnetic cores. The 45 independent observations obtained from the results of experiments conducted in the ENERCIS UKRAINE LLC company and the ‘Miskvodokanal’ of Sumy City Council contain 9 groups of 5 motor cores, ordered by increasing power, which corresponds to the variable geometry of magnetic cores. The magnetic cores of all these 4A series motors had specific losses $P_{spec} = \sim 4\ \frac{W}{kg}$ at $f = 50\ Hz$ and $B = 1\ T$, which is acceptable quality wise.

Table 2 - Relationship of specific losses in defect-free magnetic cores with T_{ec} in IMs of different size [3]

Series	P, kW	Specific losses (1/50 T/Hz)/Eddy currents time constant T_{ec}				
W22 80B4	0,75	2,97/32	3,23/41	3,78/46	3,15/51	2,9/34
W22 90S4	1,1	3,85/42	2,7/31	3,13/45	3,25/39	3,4/42
W22 90L4	1,5	2,35/26	2,9/33	3,2/41	3/38	3,2/43
W22 100L4	2,2	3,42/46	3,75/50	3/42	2,95/42	2,8/27
W22 100LB4	3,0	3,4/40	3,56/46	3,45/43	3,2/38	3,15/37
W22 112M4	4,0	3,8/43	3,65/48	3,4/47	3,63/52	3,43/39
W22 132S4	5,5	3,65/46	3,35/42	3,1/46	3,6/40	3,3/38
W22 132M4	7,5	3,9/56	3,74/44	3,2/33	3,48/48	4,1/56
W22 160L	15	3,8/53	3,45/54	3,6/47	3,2/34	3,1/30

Statistical analysis involves finding the distribution function of a given random variable x. The Gaussian (or normal) distribution has a standard density and distribution function [6]:

$$p(z) = \left(\sqrt{2\pi}\right)^{-1} e^{\frac{-z^2}{2}} \tag{21}$$

where z – random variable that looks like:

$$z = \frac{x - \mu_x}{\sigma_x} \tag{22}$$

where μ_x and σ_x – the corresponding mean and standard deviation of the random variable $x(\mathrm{T}_{ec})$.

Let’s perform the calculation using the data in table 2 and estimate the mean value of a random variable:

$$\underline{x} = \mu_x = \frac{1}{N}\sum_{i=1}^{N} x_i = 42{,}93 \tag{23}$$

The variance of a random variable:

$$\sigma_x{}^2 = \frac{1}{N}\sum_{i=1}^{N} \left(x_i - \underline{x}\right)^2 = 69{,}57 \tag{24}$$

The probability density function is shown in figure 3, and the distribution function is shown in figure 4.

As can be seen, this function almost perfectly corresponds to the Gaussian distribution function of a random variable, unimodal, monotonically varying on both sides of the mode. The fact that the above function obeys the law of normal distribution allows us to assert that the diagnostic parameter for high-quality magnetic cores is practically independent of changes in the geometry of magnetic cores.

Next, we need a parameter estimation procedure associated with the construction of an interval in which the estimated parameter T_{ec} is located with a known degree of confidence. Let the sample mean $\underline{x}$, found from N independent observations of the random variable $x(\mathrm{T}_{ec})$, be used as an estimate of the mean μ_x. Usually it is of interest to estimate μ_x in terms of some interval $x \pm \alpha$, in which μ_x falls with a given degree of confidence. Such intervals can be constructed if the sample distributions of the estimates in question are known.

It is known that the following probabilistic statement can be made about the values of the sample mean [7]:

$$Prob\left[Z_{\frac{1-\alpha}{2}} < \frac{(\underline{x} - \mu_x)\sqrt{N}}{\sigma_x} \le Z_{\frac{\alpha}{2}}\right] = 1 - \alpha \tag{26}$$

Formally, this statement is true before the sample is drawn and x is calculated. Once the sample is obtained, the value of x becomes a fairly definite number, not a random variable. Accordingly, it can be said that the probabilistic statement contained in formula (26) loses its meaning, since the value $\frac{(\underline{x} - \mu x)\sqrt{N}}{\sigma_x}$ either falls within the specified limits or does not. In other words, after obtaining the sample, the following statement is formally correct:

$$Prob\left[Z_{\frac{1-\alpha}{2}} < \frac{(\underline{x} - \mu_x)\sqrt{N}}{\sigma_x} \le Z_{\frac{\alpha}{2}}\right] = \{0, 1\} \tag{27}$$

Usually, the true value of the probability (27), which is either zero or one, is unknown. However, with the decrease of α (increase of the interval between $Z_{\frac{1-\alpha}{2}}$ and $Z_{\frac{\alpha}{2}}$), it is reasonable to assume that this probability is more likely to be one than zero. In other words, if many samples are taken and the value of x is calculated for each of them, then we can expect that the value involved in formula (27) will fall into the specified interval with a relative frequency approximately equal to $1-\alpha$. With this approach, it can be argued that there is an interval in which the value $\frac{(\underline{x}-\mu_x)\sqrt{N}}{\sigma_x}$ falls with a higher degree of confidence.

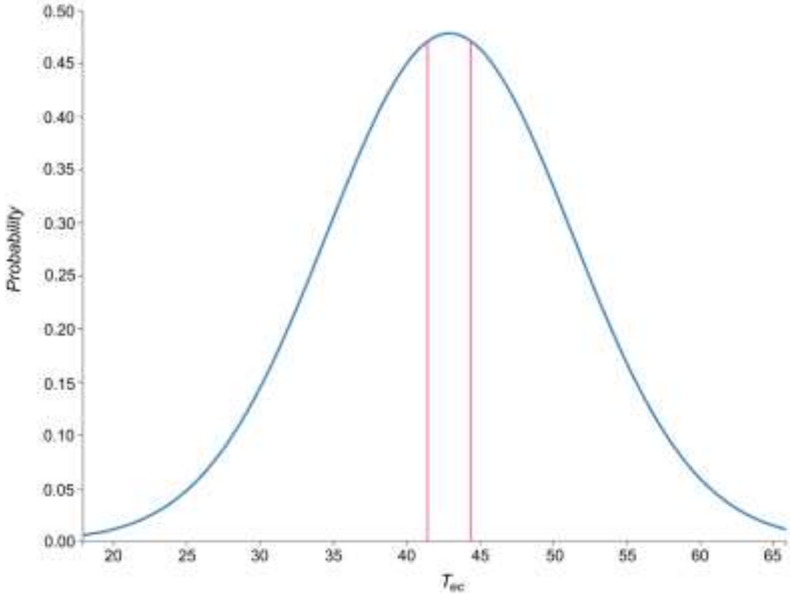

Figure 3 - *Probability density function*

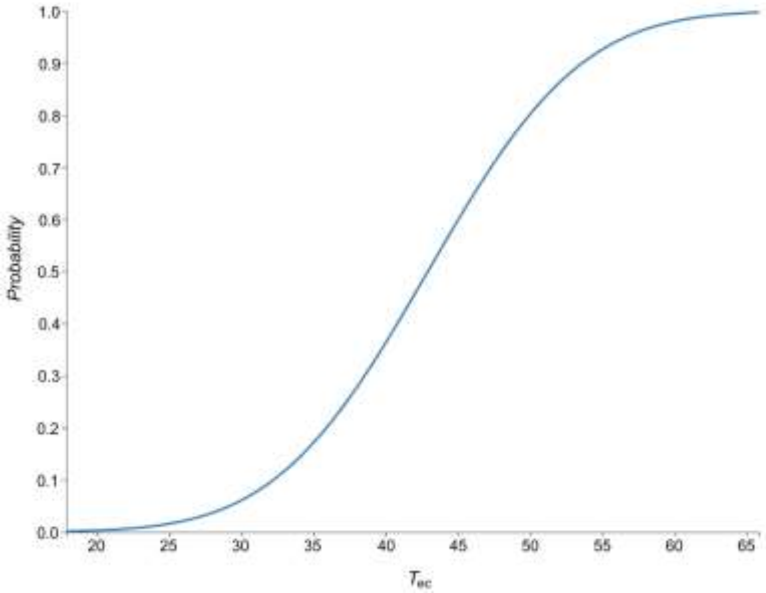

Figure 4 – *Distribution function*

Such statements are called confidence statements. The interval over which the statement is made is called the confidence interval. The degree of confidence associated with a confidence statement is called the confidence level. When estimating the mean, the confidence interval for the mean μ_x can be constructed from a sample value of $\underline{x}$ by rearranging the terms in the formula:

$$\underline{x}-\frac{\sigma_x Z_{\frac{\alpha}{2}}}{\sqrt{N}} \le \mu_x < \underline{x}+\frac{\sigma_x Z_{\frac{\alpha}{2}}}{\sqrt{N}} \quad (28)$$

If σ_x is known, then the confidence interval for μ_x can be constructed from sample values of x and S by rearranging the terms in the formula:

$$\left[\underline{x}-\frac{St_{n,\frac{\alpha}{2}}}{\sqrt{N}} \le \mu_x < \underline{x}+\frac{St_{n,\frac{\alpha}{2}}}{\sqrt{N}}\right], n = N-1 \quad (29)$$

In formulas (26, 27), the properties $Z_{1-\frac{\alpha}{2}} = -Z_{\frac{\alpha}{2}}$ are used. These intervals correspond to a confidence level of $1-\alpha$. Accordingly, the confidence statement is as follows: the true value of μ_x falls within the specified interval with a confidence level of $1-\alpha$ or (in generally accepted terms) with a confidence level of $100 * (1-\alpha)\%$. Similar statements can be made for any parameter estimates, as long as the corresponding sample distributions are known. The following formula allows us to construct a confidence interval with a confidence level of $1-\alpha$ for the variance σ_x^2 from the sample variance S^2 calculated from a sample of size N:

$$\frac{nS^2}{\chi^2_{n;\frac{\alpha}{2}}} \leq \sigma_x^2 < \frac{nS^2}{\chi}, n = N-1 \qquad (30)$$

where χ_n^2 – is the chi-squared distribution of x, $n -$ is the degree of freedom N. In our case, the sample contains $N = 45$ independent observations of the distributed random variable x (the mean μ_x for good quality magnetic cores of different geometries).

Let us find 90 percent confidence intervals for the mean and variance of a random variable x. According to formula (29), a confidence interval with a confidence level of $1-\alpha$ for the mean μ_x is calculated based on the sample mean $\underline{x}$ and variance S^2 with a sample size of $N = 45$:

$$\left[\left(\underline{x} - \frac{St_{44;\frac{\alpha}{2}}}{\sqrt{45}}\right) \leq \mu_x < \left(\underline{x} + \frac{St_{44;\frac{\alpha}{2}}}{\sqrt{45}}\right)\right] \qquad (31)$$

From the table of percentage points of the Student's t-distribution given in the source [8], we find for $\alpha = 0{,}1$; $t_{44;\frac{\alpha}{2}} = t_{44;0{,}05} = 1{,}3011$, so the interval is as follows:

$$\left[\left(\underline{x} - 0{,}194S\right) \leq \mu_x < \left(\underline{x} + 0{,}194S\right)\right] \qquad (32)$$

According to formula (30), the confidence interval for the variance σ_x^2 with confidence level $1-\alpha$ is calculated based on the sample variance S^2 with a sample size of $N = 45$:

$$\left[\frac{44S^2}{\chi^2_{44;\frac{\alpha}{2}}} \leq \sigma_x^2 < \frac{44S^2}{\chi^2_{44;\frac{1-\alpha}{2}}}\right] \qquad (33)$$

From the table of percentage points given in the source [8], the χ-squared of the distribution for $\alpha = 0{,}1$ is $\chi^2_{44;\frac{\alpha}{2}} = \chi^2_{44;0{,}05} = 56{,}37$ and $\chi^2_{44;1-\frac{\alpha}{2}} = \chi^2_{44;0{,}95} = 32{,}49$ so the interval takes the form:

$$(0{,}78S^2) \leq \sigma_x^2 < (1{,}35S^2) \qquad (34)$$

It remains to substitute the sample mean and sample variance into the formulas for the confidence intervals. Sample mean:

$$\underline{x} = \frac{1}{N}\sum_{i=1}^{N} x_i = 42{,}93 \qquad (35)$$

and, sample variance:

$$S^2 = \frac{1}{N-1}\sum_{i=1}^{N} \left(x_i - \underline{x}\right)^2 = 69{,}57 \qquad (36)$$

Thus, confidence intervals with a 90% confidence level for the mean and variance of a random variable x are as follows:

$$41{,}31 \leq \mu_x < 44{,}55 \qquad (37)$$

$$54{,}27 \leq \sigma_x^2 < 93{,}92 \qquad (38)$$

this area is shown in figure 3.

The study suggests that the distribution of high-quality magnetic cores along μ_x can be performed with a very high level of accuracy for any core geometry, provided that the μ_x value itself can be accurately determined.

Conclusions

Transient processes in circuits with ferromagnets were analysed. The analytical and statistical analysis based on the results of experiments on real samples confirms the theoretical conclusions of the possibility of estimating the specific losses in magnetic cores by the parameters of transients. A significant correlation between the specific losses in magnetic cores and the transient time constant has been established, the regression dependence has been determined, and confidence intervals have been calculated to assess the accuracy of the measurement parameters.

It is shown that the main destabilising factor affecting the reliability of the results is the failure to take into account the magnetic delay component in hysteresis phenomena, as well as the geometry of the parasitic circuit closure of eddy currents.

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АНАЛІТИЧНІ ТА СТАТИСТИЧНІ ОЦІНКИ МЕТОДИКИ ОЦІНЮВАННЯ ЯКОСТІ ШИХТОВАНИХ МАГНІТОПРОВОДІВ

В роботі вирішене актуальне питання підвищення надійності роботи електромеханічних перетворювачів з шихтованим сталевим магнітопроводом шляхом здійснення ефективних заходів по діагностуванню стану якості ламінованого осердя. Запропоновано метод оцінки якості ламінованих магнітопроводів з ізольованими листами, який ґрунтується на аналізі відклику електромагнітних взаємопов'язаних контурів на тестові швидкоплинні процеси з метою оцінки розвиненості дефектів міжлистової ізоляції та концентрації паразитних вихрових струмів, що пов'язані з питомими втратами в магнітопроводі, що є нормативним методом оцінки магнітопроводу, найбільш наближеним до роботи електричної машини в реальних умовах. Враховуючи складність та трудоємність нормативних методів контролю якості осердь обгрунтована оцінка зв'язку постійної часу загасання вихрових струмів з питомими втратами в магнітопроводі дозволить суттєво спростити методику випробувань, реалізація якої не потребує складної та дорогої апаратури та високої кваліфікації обслуговуючого персоналу. В статті крім фізичного зв'язку показано статистичний зв'язок між постійною часу загасання вихрових струмів та питомими витратами на низці реальних магнітопроводів і проведена статистична обробка результатів експериментальних даних. Отримана лінійна регресивна залежність та оцінено коефіцієнт кореляції, що підтверджує значущу залежність між наведеними вище складниками на прикладі магнітопроводів різної геометрії. Оцінено довірчі інтервали щодо бездефектних і дефектних магнітопроводів з врахуванням геометричних розмірів, потужностей та кількості полюсів досліджуваних електродвигунів.

Ключові слова: *Магнітопов'язані контури, постійна часу загасання, питомі втрати в магнітопроводі, надійність, залишковий ресурс ізоляції, шихтовані магнітопроводи*

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